Meeting 9

Math 22

Reading Debrief

- Discuss Section 10.7 w/ your group.
- Are there any questions we should address?

10.7.3.b $f_1(x,y) = 4 - x^2 - y^2,\ f_2(x,y) = x^2 + y^2,\ f_3(x,y) = x^2 - y^2.$

Compute $D = f_{xx} f_{yy} - f_{xy}^2$ at $(0,0)$ for each.

For f_1: $f_{1x} = -2x$ $\quad f_{1xx} = -2$ $\quad f_{1xy} = 0$

$f_{1y} = -2y$ $\quad f_{1yy} = -2$

$D(0,0) = f_{1xx}(0,0)\, f_{1yy}(0,0) - f_{1xy}(0,0) = 4 - 0 = 4.$

For f_2: $f_{2x} = 2x$ $\quad f_{2xx} = 2$ $\quad f_{2xy} = 0$

$f_{2y} = 2y$ $\quad f_{2yy} = 2$

$D = 4$

For f_3: $f_{3x} = 2x$ $\quad f_{3xx} = 2$ $\quad f_{3xy} = 0$

$f_{3y} = -2y$ $\quad f_{3yy} = -2$

$D = -4$

Section 10.7.2 The Second Derivative Test

Activity 10.7.4

- Complete Activity 10.7.4 and discuss w/ your group.
- Class discussion.

a. $f(x,y) = 3x^3 + y^2 - 9x + 4y$

$f_x(x,y) = 9x^2 - 9 = 0 \rightsquigarrow x = \pm 1$

$f_y(x,y) = 2y + 4 = 0 \rightsquigarrow y = -2$

$\Rightarrow$ Two critical pts $(1,-2), (-1,-2)$.

$D = f_{xx} f_{yy} - f_{xy}^2 = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{xy} & f_{yy} \end{vmatrix}$

$f_{xx}(x,y) = 18x$ $\quad f_{yy}(x,y) = 2$ $\quad f_{xy}(x,y) = 0$

$D = 36x - 0. \Rightarrow D(1,-2) = 36 > 0$

$D(-1,-2) = -36 < 0 \rightsquigarrow$ Saddle point

$\rightsquigarrow D(1,-2) > 0 \Rightarrow (1,-2)$ is a local extrema

Check $f_{xx}(1,-2) = 18 > 0 \Rightarrow$ local min.

Activity 10.7.5

- Complete Activity 10.7.6 and discuss w/ your group.
- Class discussion.

a. $R(p_1,p_2) = p_1(150 - 2p_1 - p_2) + p_2(200 - p_1 - 3p_2)$
$= 150p_1 + 200p_2 - 2p_1p_2 - 2p_1^2 - 3p_2^2.$

$R_{p_1} = 150 - 2p_2 - 4p_1 = 0$

$R_{p_2} = 200 - 2p_1 - 6p_2 = 0$

Elimination or Substitution $\to (25,25)$ is the only critical pt.

b. $R_{p_1p_1} = -4$ $\quad R_{p_1p_2} = -2$ $\quad R_{p_2p_2} = -6$

$D = \begin{vmatrix} -4 & -2 \\ -2 & -6 \end{vmatrix} = 20 > 0$

Conclusion: $D > 0 \Rightarrow (25,25)$ is a local extrema

$R_{p_1p_1} = -4 < 0 \Rightarrow (25,25)$ is a local max.

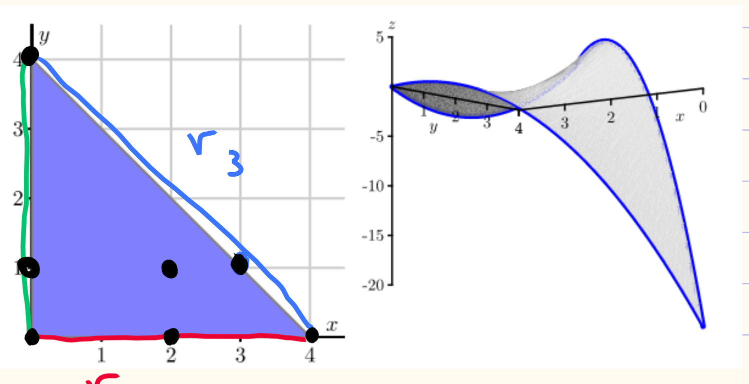
c. Set $p_1 = 25$ and $p_2 = 25$ to maximize revenue.

Section 10.7.3 Optimization on a Restricted Domain

Activity 10.7.6

- Complete Activity 10.7.6 and discuss w/ your group.
- Class discussion.

The function was $f(x,y) = x^2 - 3y^2 - 4x + 6y$.



$(0,0)$

$(4,0)$

a. Critical pts in the interior: $f_x = 0$ and $f_y = 0$

There is one: $(2,1)$.

Critical pts on the boundary: parameterize each edge of the triangle

$r_1(t)$: $x(t) = 4t,\ y(t) = 0$

$f(x,y) = x^2 - 3y^2 - 4x + 6y$ $\quad r_2(t)$: $x(t) = 0,\ y(t) = 4t$

$r_3(t)$: $x(t) = 4t,\ y(t) = 4 - 4t$

$r_1(t)$: $f(x(t), y(t)) = 16t^2 - 16t$

$32t - 16 = 0 \Rightarrow t = 1/2 \Rightarrow r_1(1/2) = (2,0)$

$\Rightarrow (2,0)$ is a critical pt.

$r_2(t)$: $f(0,4t) = -48t^2 + 24t$

$-96t + 24 = 0 \Rightarrow t = \frac{24}{96} = \frac{1}{4}$.

$r(1/4) = (0,1)$. $\Rightarrow (0,1)$ is a critical pt

$r_3(t)$: $f(4t, 4-4t) = 16t^2 - 3(4-4t)^2 - 16t + 6(4-4t)$

$32t - 6(4-4t)\cdot(-4) - 16 - 24t = 0$

$\Rightarrow t = \frac{3}{4}$ $\quad r(3/4) = (3,1)$

$\Rightarrow (3,1)$ is a critical point.

e. We found 7 critical points. To find global max/min just evaluate $f(x,y)$ at all 7 critical points and choose the largest/smallest value.